

## ON MULTIPLE POISSON INPUT QUEUE WITH ERLANGIAN SERVICE TIME DISTRIBUTION

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### ABSTRACT

A queuing model, in which the probability of occurrence of more than one arrival in a small interval of time  $\Delta t$  allowed, is considered here. The inter arrival times follow poisson distribution and the service times follow Erlangian distribution. The queuing discipline is FCFS. The explicit expression for the expected number of phases is derived using the generating function technique.

### INTRODUCTION

In queuing theory, generally the probability of occurrence of more than one arrival in a small interval of time  $\Delta t$  is not allowed. In most of the practical situations occurrence of more than one arrival is possible. In a restaurant, singlets or doublets or triplets may come for service at a time. Taking these three possibilities into consideration, a queuing model with Erlangian service is derived. Let the number of single arrivals follow Poisson distribution with mean  $\lambda_1 t$  while the number of doublets follow poisson distribution with

mean  $h_2 t$ . Let the number of triplets also follow poisson distribution with mean  $\lambda_3 t$ . The occurrence of singlets, doublets and triplets are independent of each other. The queuing discipline is first come first served. The customers arrive towards the service facility with an infinite capacity and each source arrival creates  $k$  phases of service, that is, each source arrival adds to the number of phases in the system by  $k$ .

### MODEL FORMULATION

Let  $P_n$  be the probability that there are  $n$  phases in the system. The probability that a new customer is added in the interval of length  $\Delta t$  is  $\lambda_1 \Delta t + O(\Delta t)$  while the probability that two customers are added in the interval  $\Delta t$  is  $\lambda_2 \Delta t + O(\Delta t)$ . Similarly the probability that three customers arrive in the interval  $\Delta t$  is  $\lambda_3 \Delta t + O(\Delta t)$ . The difference-differential equations for the queuing process are

$$\frac{d P_0(t)}{dt} = -(\lambda_1 + \lambda_2 + \lambda_3) P_0(t) + k\mu P_1(t) \quad (1)$$

$$\frac{d P_n(t)}{dt} = -(\lambda_1 + \lambda_2 + \lambda_3 + k\mu) P_n(t) \quad (2)$$

$$+ \lambda_1 P_{n-k}(t) + \lambda_2 P_{n-2k}(t) + \lambda_3 P_{n-3k}(t) + k\mu P_{n+1}(t)$$

$$n \geq 1$$

After the steady state is assumed for the queuing process, the steady state equations are

$$(\lambda_1 + \lambda_2 + \lambda_3) P_0 = k\mu P_1 \quad (3)$$

$$(\lambda_1 + \lambda_2 + \lambda_3 + k\mu) P_n = \lambda_1 P_{n-k}$$

$$+ \lambda_2 P_{n-2k} + \lambda_3 P_{n-3k} + k\mu P_{n+1} \quad (4)$$

$$n \geq 1$$

The probabilities having negative subscripts are zero.

## MEAN NUMBER OF UNITS IN THE SYSTEM

Let us define probability generating functions as

$$P(Z) = \sum_{n=0}^{\infty} P_n z^n, \quad z \leq 1 \quad (5)$$

Multiplying equation (4) by  $z^n$  and summing over  $i = 1, 2, \dots, \infty$  and then adding (3), we get

$$\begin{aligned} & (\lambda_1 + \lambda_2 + \lambda_3 + k\mu) \sum_{n=k}^{\infty} P_n z^n + (\lambda_1 + \lambda_2 + \lambda_3) P_0 \\ &= \lambda_1 z^k \sum_{n=k}^{\infty} P_{n-k} z^{n-k} + \lambda_2 z^{2k} \sum_{n=2k}^{\infty} P_{n-2k} z^{n-2k} \\ &+ \lambda_3 z^{3k} \sum_{n=3k}^{\infty} P_{n-3k} z^{n-3k} + \frac{k\mu}{z} P_1 z \\ &+ \frac{k\mu}{z} \sum_{n=1}^{\infty} P_{n+1} z^{n+1} \end{aligned} \quad (6)$$

After simplifying (6), we get

$$\begin{aligned} & [(1-z) - S_1 z(1-z^k) - S_2 z(1-z^{2k}) - S_3 z(1-z^{3k})] P(z) \\ &= (1-z) P_0 \end{aligned} \quad (7)$$

Dividing equation (7) by  $(1-z)$ , we get

$$\begin{aligned} & [1 - S_1 z(z^{k-1} + z^{k-2} + \dots + z^1 + z^0) \\ & - S_2 z(z^{2k-1} + z^{2k-2} + \dots + z^1 + z^0) \\ & - S_3 z(z^{3k-1} + z^{3k-2} + \dots + z^1 + z^0)] P(z) \\ &= P_0 \end{aligned} \quad (8)$$

Putting  $z=1$  and  $P(1) = 1$  in (8), we get

$$P_0 = 1 - k(S_1 + 2S_2 + 3S_3) \quad (9)$$

$$P(z) = \frac{P_0}{1 - S_1(z^k + z^{k-1} + \dots + z) - S_2(z^{2k} + z^{2k-1} + \dots + z) - S_3(z^{3k} + \dots + z)} \quad (10)$$

The expected number of phases is given by

$$L = P'(z) |_{z=1} = \frac{P_0 [k(k+1)/2 + 2k(2k+1)/2 + 3k(3k+1)/2]}{[1 - k(S_1 + 2S_2 + 3S_3)]^2} \quad (11)$$

The variance and higher moments of the system can be calculated very easily.

#### REFERENCES

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